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Indirect Causes in Dynamic Bayesian Networks Revisited

Abstract Modeling causal dependencies often demands cycles at a coarse-grained temporal scale. If Bayesian networks are to be used for modeling uncertainties, cycles are eliminated with dynamic Bayesian networks, spreading indirect dependencies over time and enforcing an infinitesimal resolution of time. Without a "causal design," i.e., without anticipating indirect influences appropriately in time, we argue that such networks return spurious results. By introducing activator random variables we propose template fragments for modeling dynamic Bayesian networks under a causal use of time, anticipating indirect influences on a solid mathematical basis, obeying the laws of Bayesian networks.

Introduction

In a company, we are concerned with regulatory compliance over time. **Manipulated** and **exchanged documents** might **influence employees** becoming credulous at time t, who, further, might influence other employees. We represent the credulousness state of an employee as a random variable C^tlaire, D^ton, and E^tarl, and message-exchange variables from X to Y at t as M^t_{XY}.

Say, influences only occur from C to D to E. Fig. 1 correctly represents this as a classic DBN.

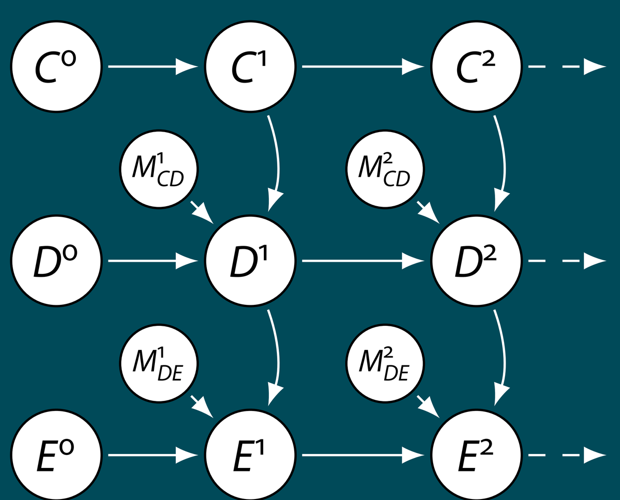

Figure 1: A simple DBN.

Now, **say every employee can influence everyone**. One now has two options for modeling this problem as shown in Fig. 2: using inter-dependencies (dashed "diagonal") or using intra-dependencies (thick "cyclic").

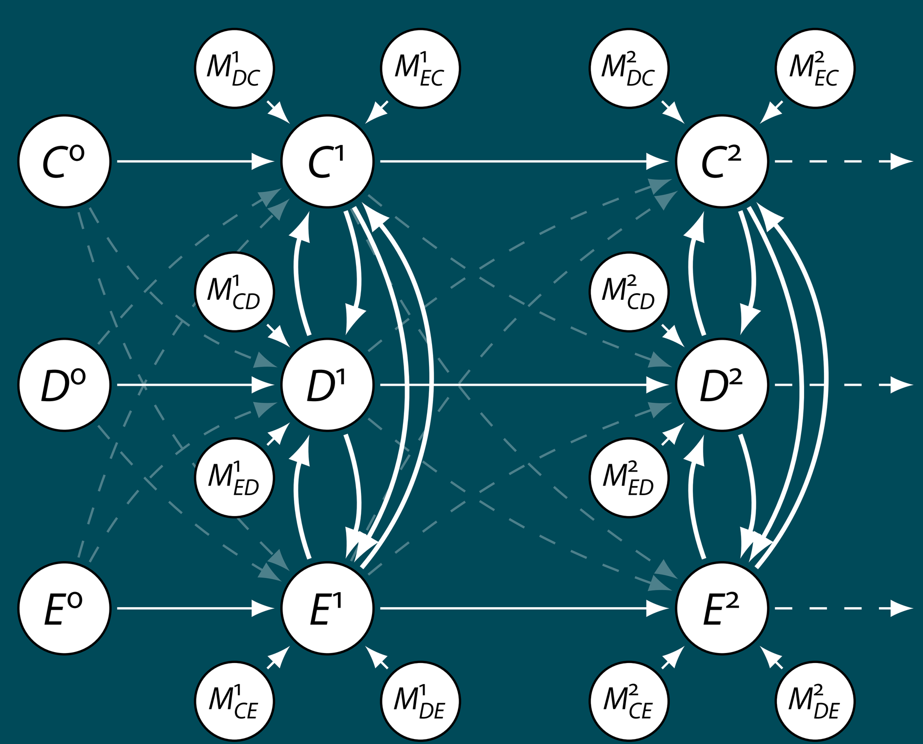

Figure 2: Everyone can influence everyone. Thick contains cycles. Dashed "diagonal" is causally incorrect under an open use of time.

Intuitive and causally correct intra-dependencies (thick) create **forbidden cycles** and inter-dependencies ("diagonal", dashed) in the style of HMM are the only alternative. Unfortunately, inter-dependencies heavily restrict DBNs in their expressiveness and usability:

Proposition 1 (DBN Restrictions)**. In classic DBNs**, indirect influences are spread over multiple timesteps and possible **indirect influences cannot be considered** during one timeslice. This enforces **a)** an **infinitesimal resolution of observations** or **b) restricts** a DBN to **observations** where indirect influences strictly do not occur. This implies, no two activators M^t_{*i} and M^t_{i*} can be probably active. ▲

Activator DBNs

Message-exchanges M^t_{XY} are seen as **activator random variables**.

Definition 1 (Activator Random Variable)**.** A_{XY} is an activator random variable which activates a dependency of random variable Y on X in a given context. Let $\mathrm{dom}(A_{XY}) = \{true, false\}$. We define the *deactivation* criterion from a functional perspective towards the CPT as

$$\forall x, x' \in \mathrm{dom}(X), \forall y \in \mathrm{dom}(Y), \forall \vec{z} \in \mathrm{dom}(\vec{Z}) :$$
$$P(y|x, \neg a_{XY}, \vec{z}) = P(y|x', \neg a_{XY}, \vec{z}) = P(y|*, \neg a_{XY}, \vec{z}) ,$$

where $*$ represents a wildcard and $\vec{z}$ further dependencies. ▲

Both options in Fig. 2 are supported in **Activator Dynamic Bayesian Networks** (ADBN).

Definition 2 (ADBN)**.** An ADBN fragment template $B'_{\rightarrow}$ consists of dependencies between states X^s_i and X^t_j, $t-1 \leq s \leq t$ (Markov-1) and matrices $A^{s\,t}$ of activators. Let $A^{s\,t}_{ij}$ be the activator random variable influencing X^t_j regarding a dependency on X^s_i, such that X^t_j's local CPT follows Def. 1. Every activator is assigned a prior probability. An ADBN is then syntactically defined by $(B_0, B'_{\rightarrow})$ defining its semantics as a well-defined joint probability $P(\vec{X}^{0:t^\top}, \vec{\mathcal{A}}^{01:tt^\top})$. ▲

In fact, we show that **ADBNs can be based on cyclic graphs** under much softer restrictions and anticipate indirect influences under an open use of time.

Theorem 1 (Bayesian Network Soundness)**.** For every set of instantiations $\vec{\mathcal{A}}^{1:t}_*$ **an ADBN corresponds to a Bayesian network** (BN), if for all t, $\vec{\mathcal{A}}^t_*$ satisfies a new acyclicity constraint:

$$\forall x, y, z \in \vec{X}^t : \mathfrak{A}(x,z)^t, \mathfrak{A}(z,y)^t \rightarrow \mathfrak{A}(x,y)^t$$
$$\neg\exists q : \mathfrak{A}(q,q)^t ,$$

with a function $\mathfrak{A}(i,j)^t$ that is defined as

$$\mathfrak{A}(i,j)^t = \begin{cases} false & \text{if} \quad A^t_{ij} = \neg a^t_{ij} \\ true & \text{otherwise} \end{cases} .$$

Given a correspondence to a BN an ADBN's semantics is well-defined and the joint probability over all variables is specified by,

$$P(\vec{X}^{0:t^\top}, \vec{\mathcal{A}}^{1:t^\top}) = P(\vec{X}^{0:t-1^\top}, \vec{\mathcal{A}}^{1:t-1^\top}) \cdot \prod_i P(X^t_i | \vec{X}^{t^\top} \setminus X^t_i, \vec{A}^{t^\top}_i, X^{t-1}_i) \cdot P(\vec{\mathcal{A}}^{t^\top}) . \quad ▲$$

Proof of Thm. 1 is given in our paper.

Operations

Given observations $\vec{z}^{\,t}, \vec{\mathfrak{b}}^t$, i.e., (partial) instantiations of $\vec{X}^t, \vec{\mathcal{A}}^t$, under which Thm. 1 is obeyed, usual operations for DBNs are well-defined even in cyclic ADBNs.

Definition 3 (Filtering)**.**

$$P(\vec{X}^{t^\top}, \vec{\mathcal{A}}^{t^\top} | \vec{z}^{\,0:t^\top}, \vec{\mathfrak{b}}^{1:t^\top}) = \alpha \cdot \sum_{\vec{\zeta}^{t-1^\top}} \sum_{\vec{\beta}^{t-1^\top}} P(\vec{X}^{t-1^\top}, \vec{\mathcal{A}}^{t-1^\top} | \vec{z}^{\,0:t-1^\top}, \vec{\mathfrak{b}}^{1:t-1^\top}) \cdot \prod_i P(X^t_i | \vec{X}^{t^\top} \setminus X^t_i, \vec{A}^{t^\top}_i, X^{t-1}_i) \cdot P(\vec{\mathcal{A}}^{t^\top}) . \quad ▲$$

As usual, filtering from $t-1$ to t has time and space complexity $\mathcal{O}(1)$.

Definition 4 (Smoothing)**.**

$$P(\vec{X}^{k^\top}, \vec{\mathcal{A}}^{k^\top} | \vec{z}^{\,0:t^\top}, \vec{\mathfrak{b}}^{1:t^\top}) = \alpha \cdot P(\vec{X}^{k^\top}, \vec{\mathcal{A}}^{k^\top} | \vec{z}^{\,0:k^\top}, \vec{\mathfrak{b}}^{1:k^\top}) \cdot \sum_{\vec{\zeta}^{k+1^\top}} \sum_{\vec{\beta}^{k+1^\top}} \prod_i P(X^{k+1}_i | \vec{X}^{k+1^\top} \setminus X^{k+1}_i, A^{k+1^\top}_i, X^k_i) \cdot P(\vec{\mathcal{A}}^{k+1^\top}) \cdot P(\vec{z}^{\,k+2:t^\top}, \vec{\mathfrak{b}}^{k+2:t^\top} | \vec{X}^{k+1^\top}, \vec{\mathcal{A}}^{k+1^\top}) . \quad ▲$$

As usual, smoothing over all $k < t$ has $\mathcal{O}(t^2)$ time and constant space complexity or, by storing filtering operations, $\mathcal{O}(t)$ time and space complexity.

Contributions

We show that **DBNs pose conflicts with causality when indirect effects need to be anticipated** and enforce high-frequent updates of observations. Further, by introducing ADBNs we have shown that **(A)DBNs can** actually **be based on cyclic graphs** under much softer restrictions **sound to Bayesian networks**. ADBNs **causally correctly anticipate indirect causes** in DBNs under an **open choice of time granularity**.

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